

# Recent study on the conformal fixed point in the SU(3) gauge theory

arXiv:1212.1353[hep-lat] and work in progress

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2013/02/07@ BNL

# Summary of our work

- Study SU(3) gauge theory coupled with 12 massless fundamental fermions
- Measured the running coupling constant in Twisted Polyakov loop scheme and found an IR fixed point
- Derived the universal quantities around the fixed point
  - critical exponent of the beta fn.
  - mass anomalous dimension

# Introduction

Higgs sector in the Standard Model Lagrangian

$$\mathcal{L}_H \sim \frac{1}{2} D_\mu \phi D^\mu \phi^\dagger + \frac{\lambda}{4} (\phi \phi^\dagger - v^2)^2$$

Problem with a fundamental Higgs boson

Hierarchy problem (need fine-tuning to cancel a quadratic divergence)

Triviality problem



$$\beta(\lambda) = \frac{3\lambda^2}{2\pi^2} > 0$$

No interaction at low energy

Running coupling constant diverges at a finite energy

Cutoff theory?

Candidates for the origin of Higgs sector

Supersymmetry

Extra dimension

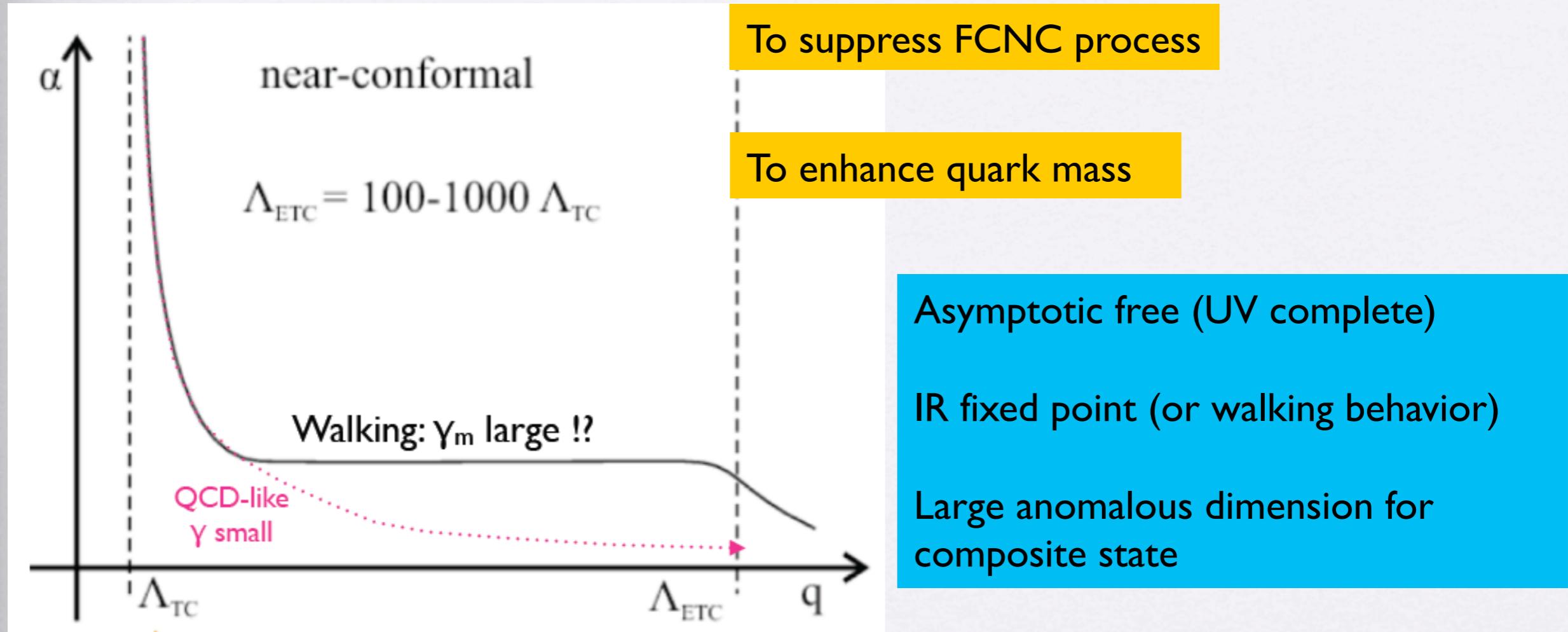
Walking techni-color

Fourth generation

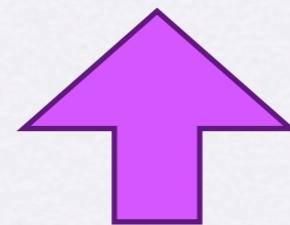
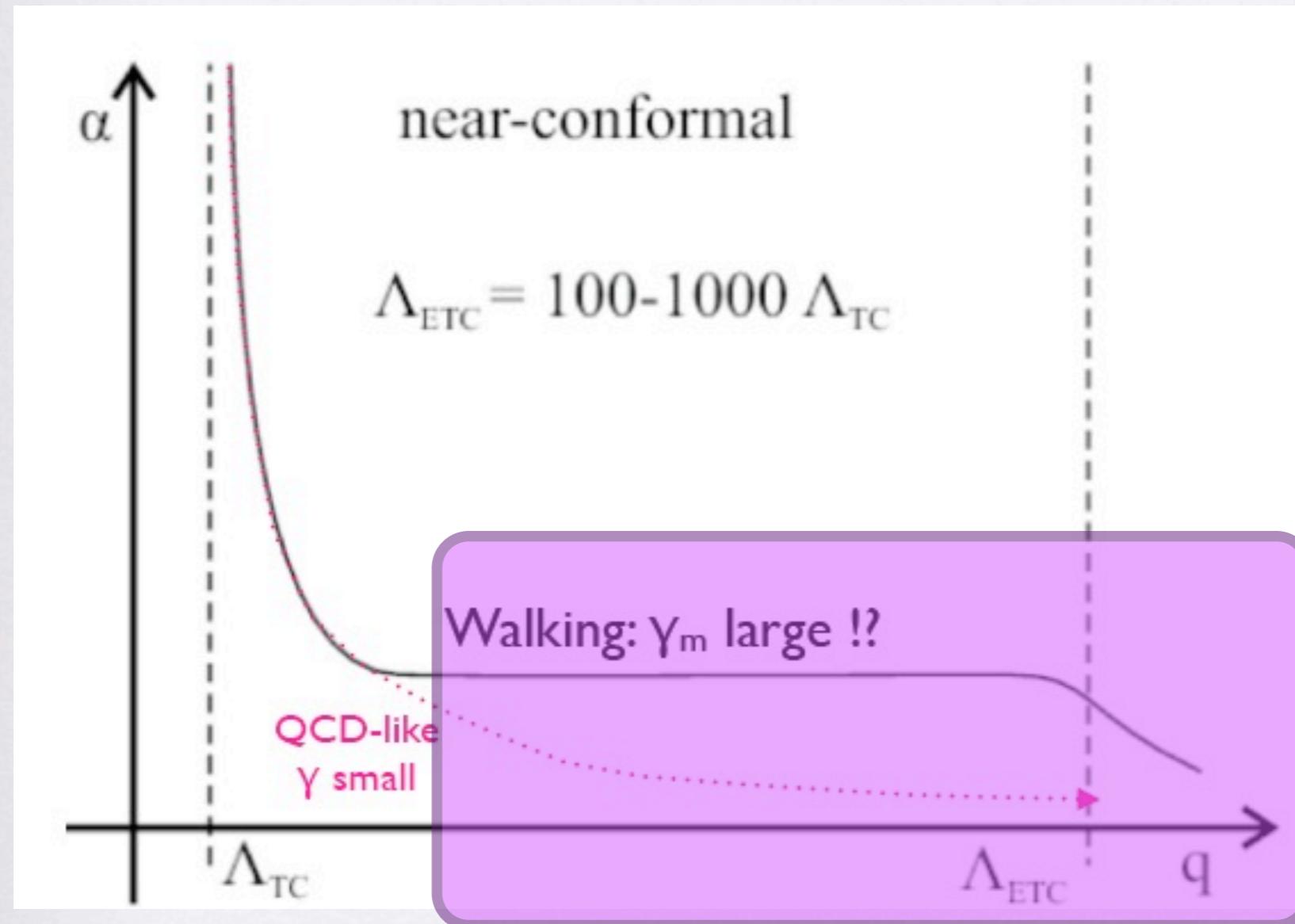
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Introduce an additional gauge interaction  
and fermions

# Walking Technicolor



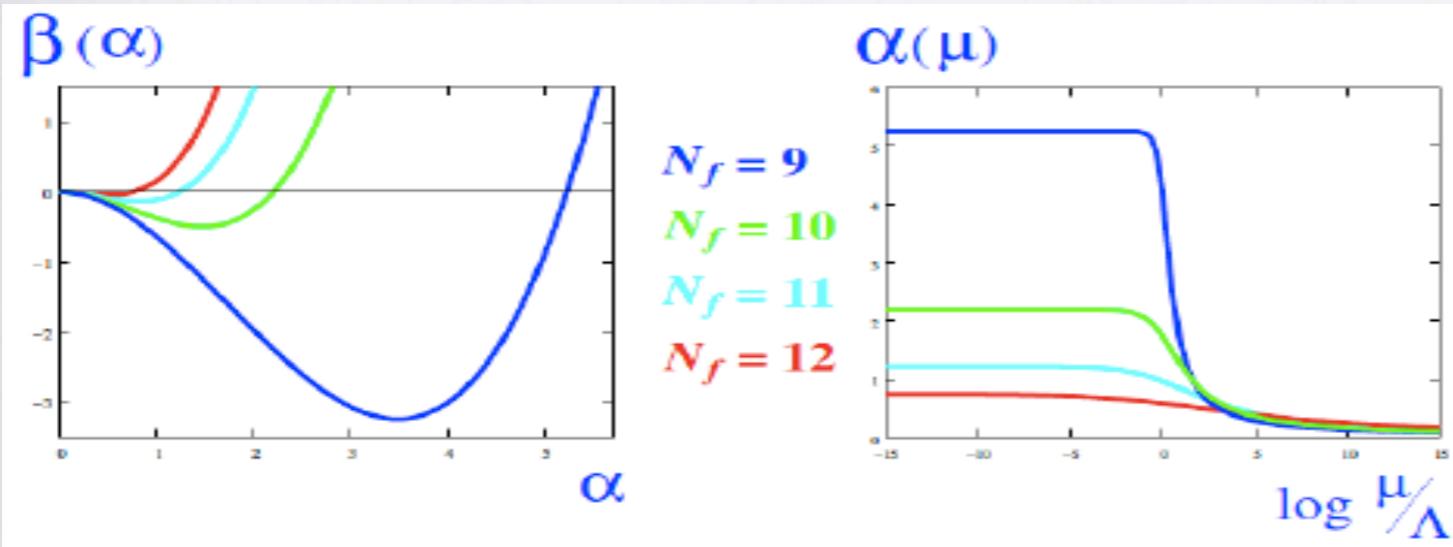
Is there a theory whose coupling constant show the behavior?



This part may be realized by  
many flavor gauge theory.

# SU(3) Nf=12 theory

Two loop analysis



Phase structure based on two loop



**perturbative (MS bar scheme)**

2-loop	3-loop	4-loop
(alpha) 0.75	0.44	0.47
(g^2) 9.4	5.5	5.9

T.A.Ryttov and R.Shrock,  
Phys.Rev.D83,056011 (2011)

**S-D eq. with large Nc**

$$N_f^{cr} = 11.9$$

**Exact RG**

$$N_f^{cr} = 10.0^{+1.6}_{-0.7}$$

H.Gies and J.Jaeckel,  
Eur.Phys.J. G46:433-438,2006

**Exact RG (+ 4 fermi interaction)**

$$N_f^{cr} = 11.58$$

Y.Kusafuka and H.Terao,  
arXiv:1104.3606 [hep-ph]

# Is there an IR fixed point in SU(3) Nf=12 theory?

Iwasaki et al, '04 '13 (phase structure)

Appelquist, Fleming, Neil '07, '09, '11 (running coupling, mass spectrum)

Deuzeman, Lombardo, Pallante, Miura '09, '11(finite temperature)

A. Hasenfratz '09, '10 (MCRG, phase structure)

DeGrand '11 (mass spectrum)

LatKMI '12 (mass spectrum)

Fodor et al. '09 , '11(running coupling, phase structure, spectrum)

Jin and Mawhinney '09 (phase structure)

Why there are  
controversial results?

# Why there are controversial results?

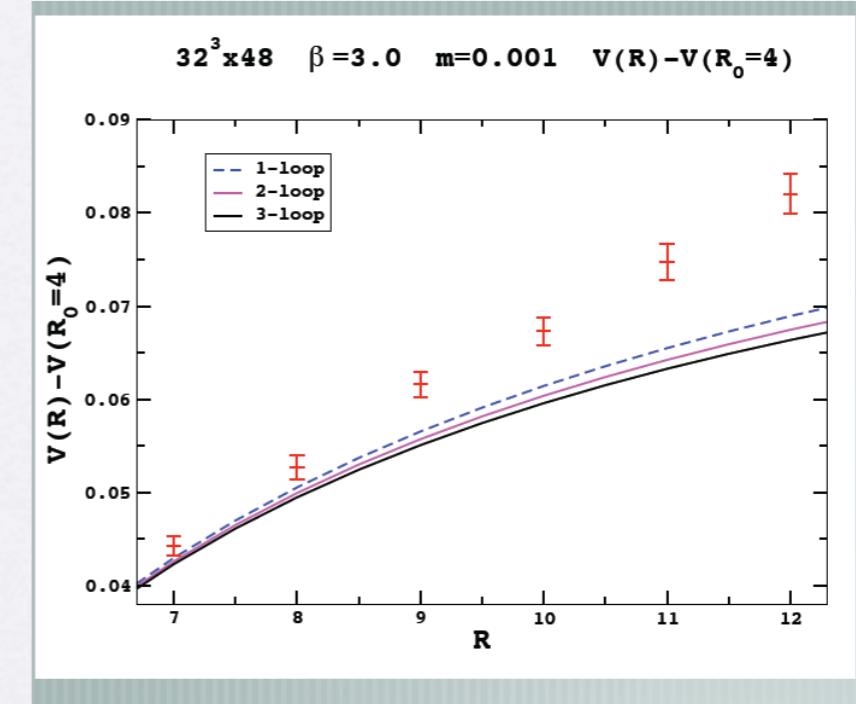
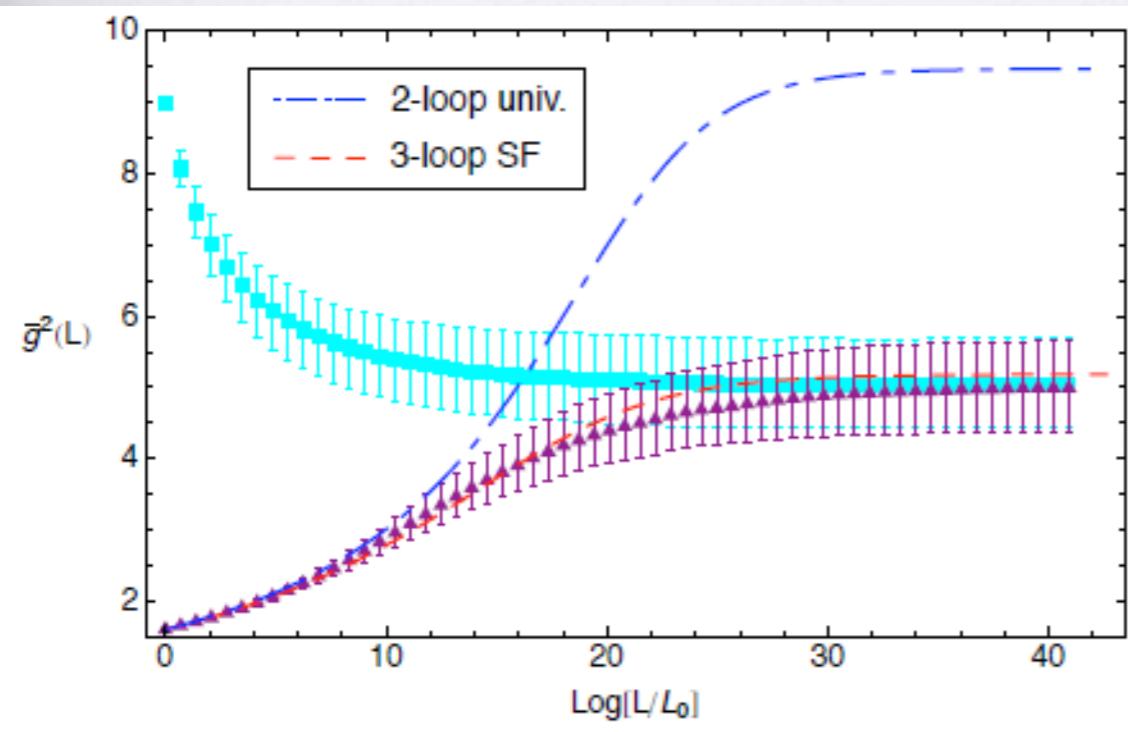
The continuum extrapolation should be taken carefully.  
(conformal theory is realized in the continuum)

# Present status on the study of IR behavior in the case of SU(3) Nf=12

## Running coupling constant

Appelquist et al. (SF scheme)  
Phys. Rev. D79:076010, 2009

Fodor et al. (potential scheme)  
PoS LAT2009:055, 2009,  
talk at Lattice2010



Plot: Slide of K.Holland's talk at Lattice2010

## Relationship between two renormalization schemes

scheme transformation:

$$g_1 \rightarrow g_2 = f(g_1)$$

$f(g_1)$  is an analytic fn. of  $g_1$

beta fn.  $\beta(g_2) = \frac{\partial f(g_1)}{\partial g_1} \beta(g_1)$

The existence of the fixed point is scheme independent.

Note that the renormalized coupling constant at the fixed point depends on the scheme.

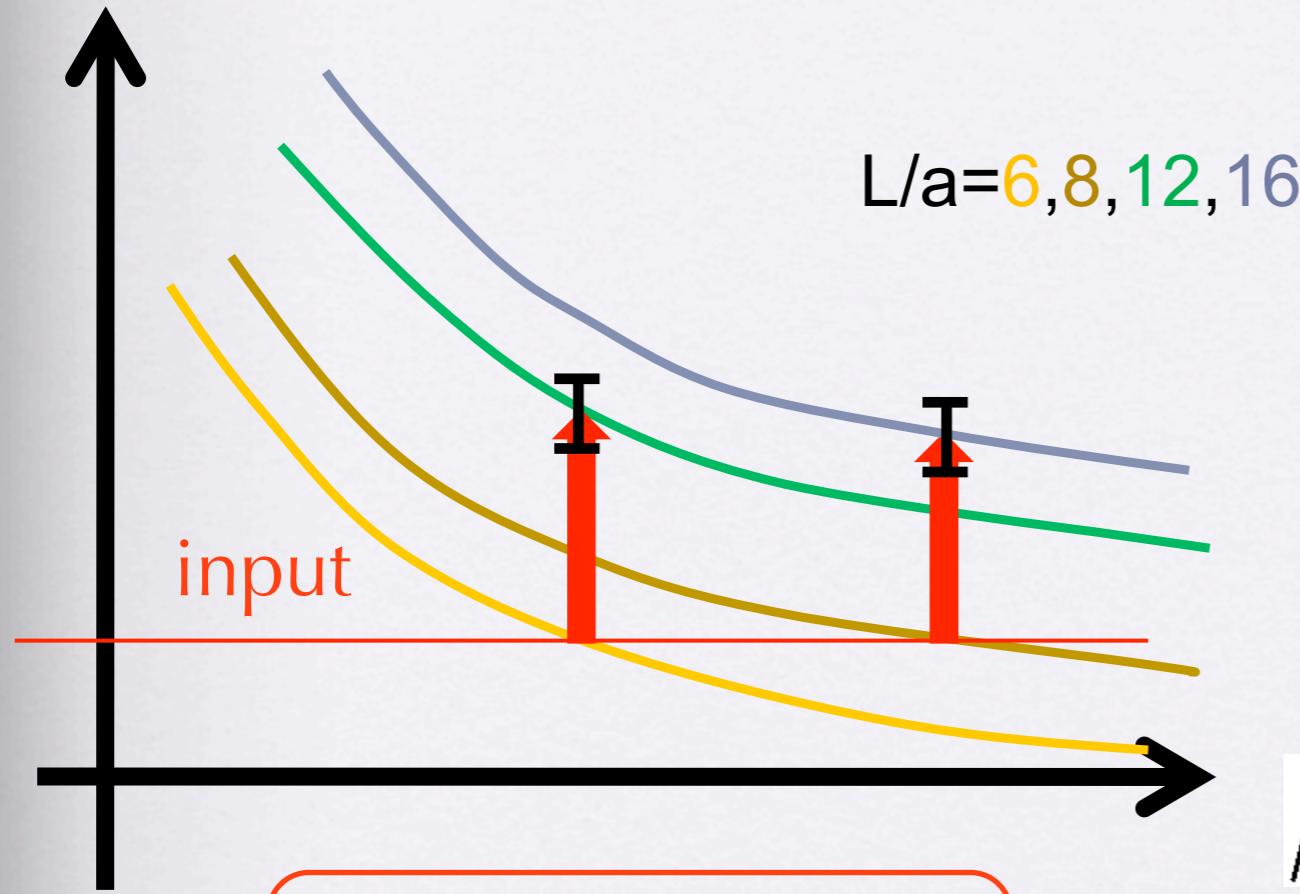
$$g_2^* = f(g_1^*)(\neq g_1^*)$$

# Step scaling method

- measure the running coupling constant -

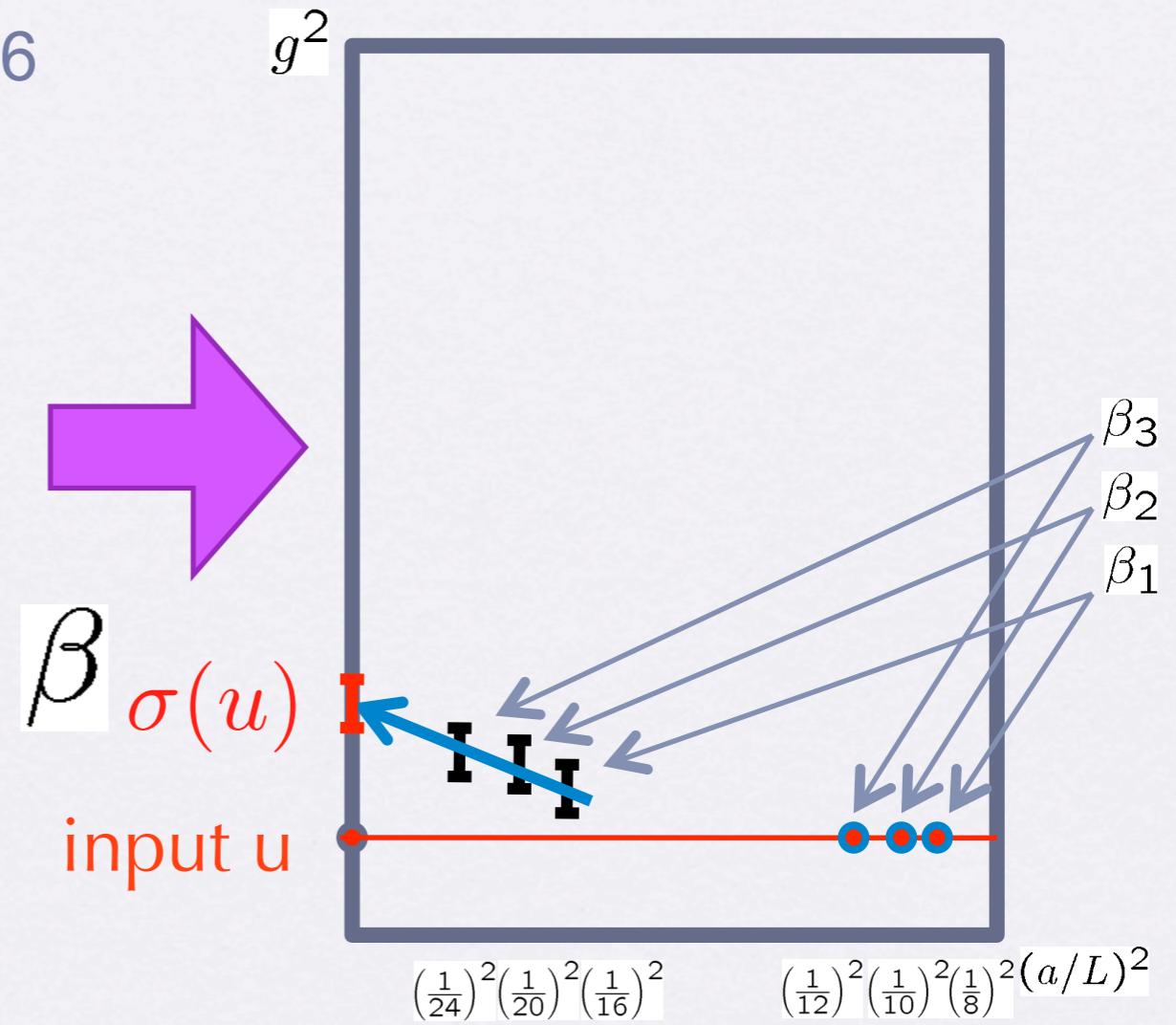
Luescher, Weisz and Wolff, NPB 359 (1991) 221

$$g_R^2(\beta, L/a)$$



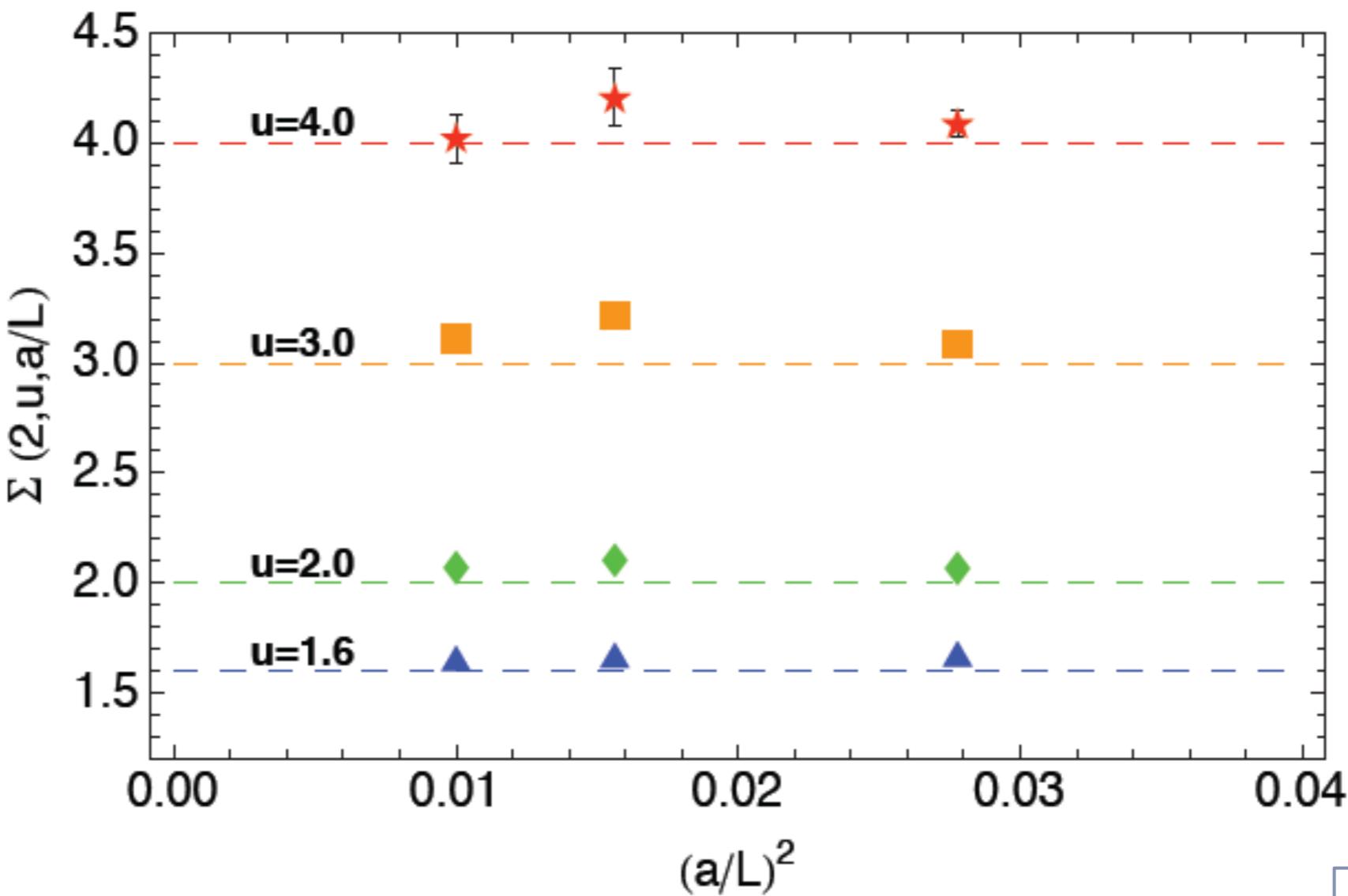
$$\sigma(u) \equiv g_R^2(1/sL)$$
$$u \equiv g_R^2(1/L)$$

- tune beta to reproduce the input renormalized coupling
- measure the  $g^2$  at the larger lattice with the tuned beta
- take the continuum limit



# Present status on the study of IR behavior in the case of SU(3) Nf=12

constant extrapolation?



s=2 step scaling  
 $L=6 \rightarrow L=12$   
 $L=8 \rightarrow L=16$   
 $L=10 \rightarrow L=20$

# Why there are controversial results?

The continuum extrapolation should be taken carefully.  
(conformal theory is realized in the continuum)

In the study on the phase structure, parameter search is not enough?  
(tuning the beta value)

# chiral symmetry

Z.Fodor et al.: Phys.Lett.B703:348-358,2011.

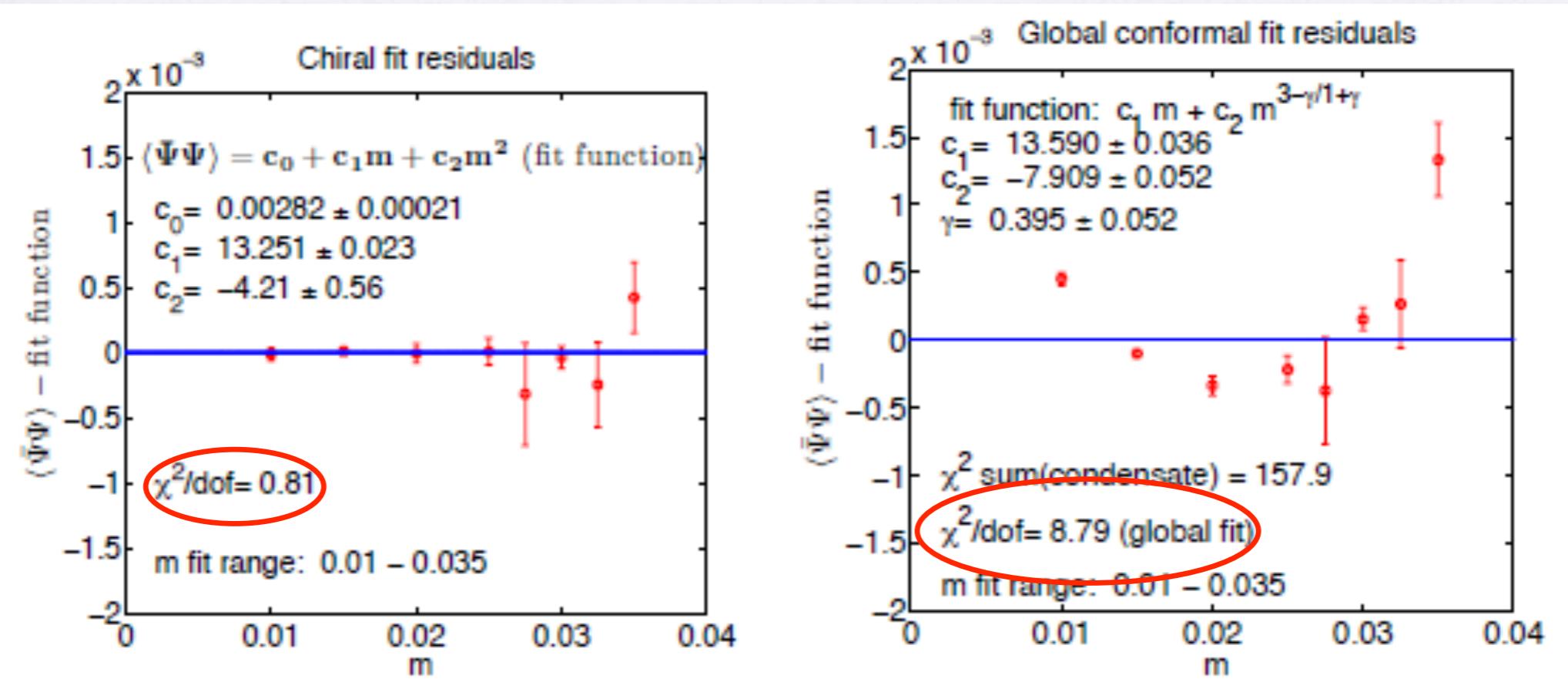
measured mass spectrum and chiral condensate at beta=2.2  
in several lattice sizes and fermion bare masses.

## Comparison between two hypotheses

The vertical axis denotes

$$\langle \bar{\Psi} \Psi \rangle - (c_0 + c_1 m + c_2 m^2)$$

$$\langle \bar{\Psi} \Psi \rangle - (c_1 m + c_2 m^{3-\gamma/(1+\gamma)})$$



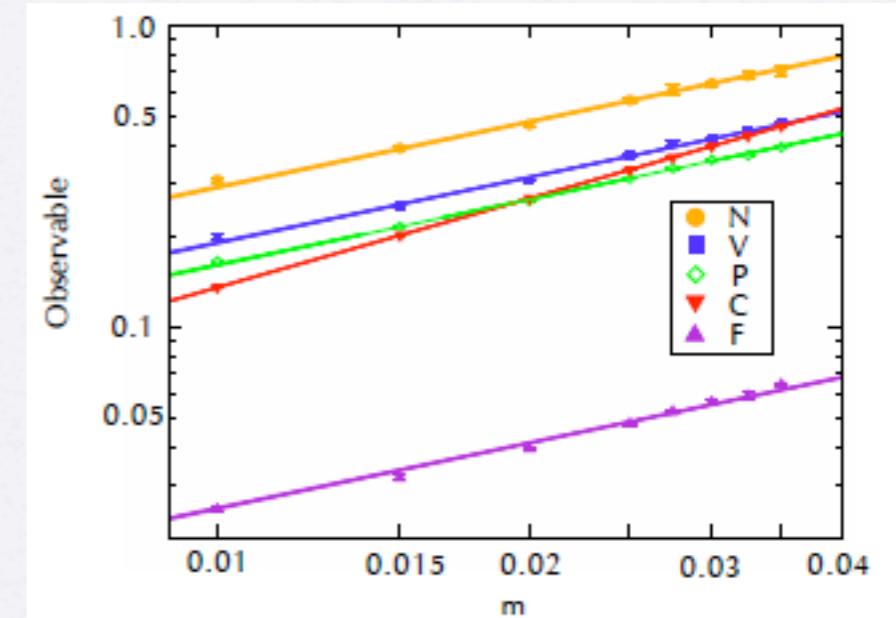
chiral extrapolation using conformal hypothesis does not work.

Appelquist's group PRD84(2011)054501

Fit only the largest lattice data with each bare mass.

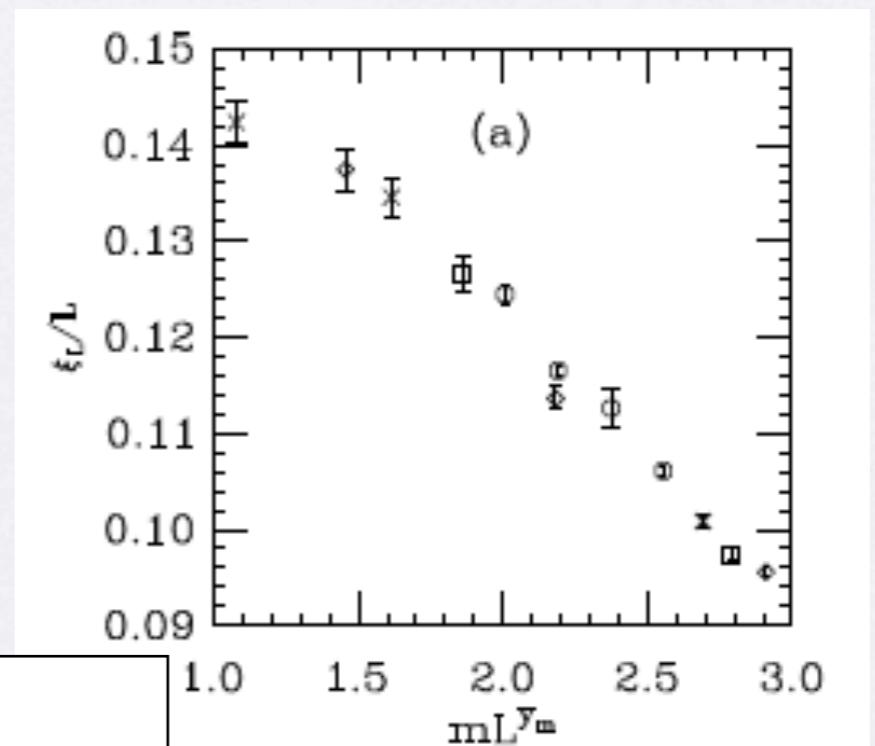
$$M_X \sim m_q^{\frac{1}{1+\gamma_m^*}} + m_q$$

DeGrand PRD84 (2011) 116901



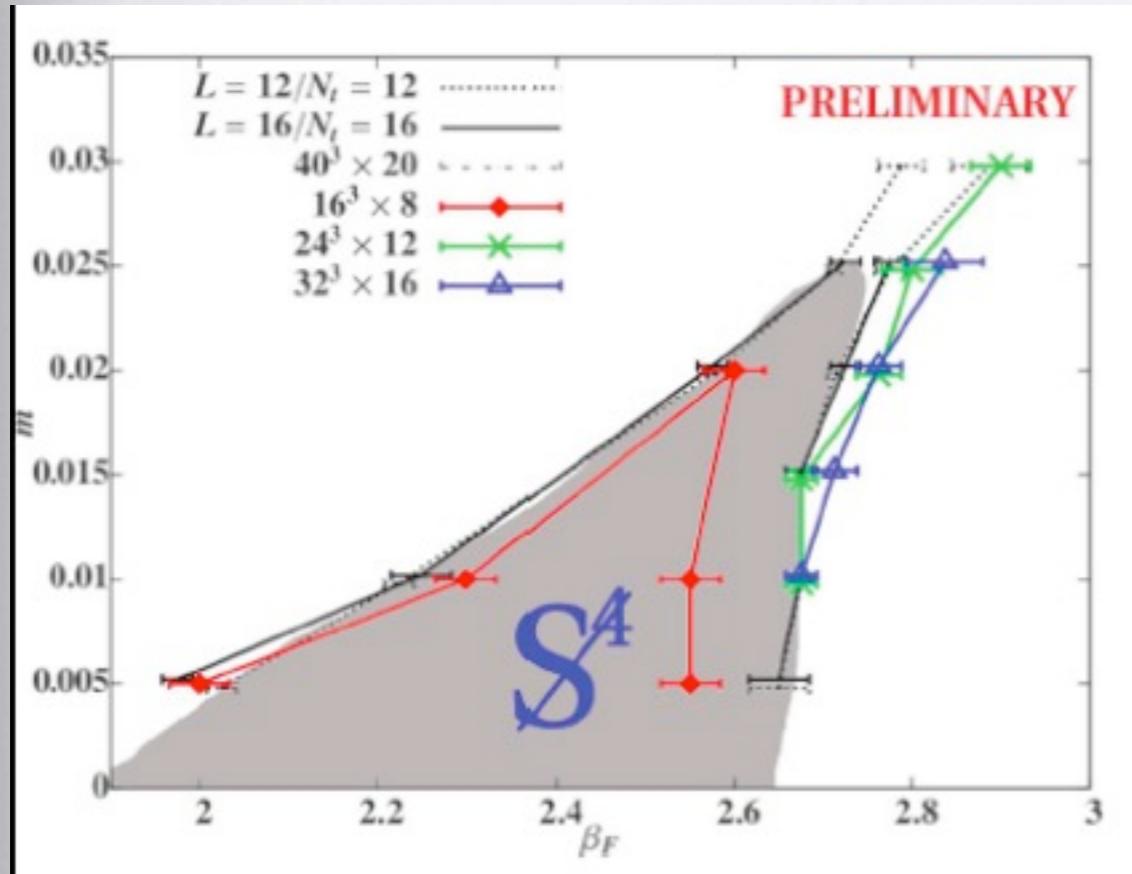
Fit the data under consideration with finite-scaling effect

$$\frac{1}{M_X L} = F(m_q L^{1+\gamma_m^*})$$



conformal hypothesis also works well  
similar fit quality with chiral broken hypothesis

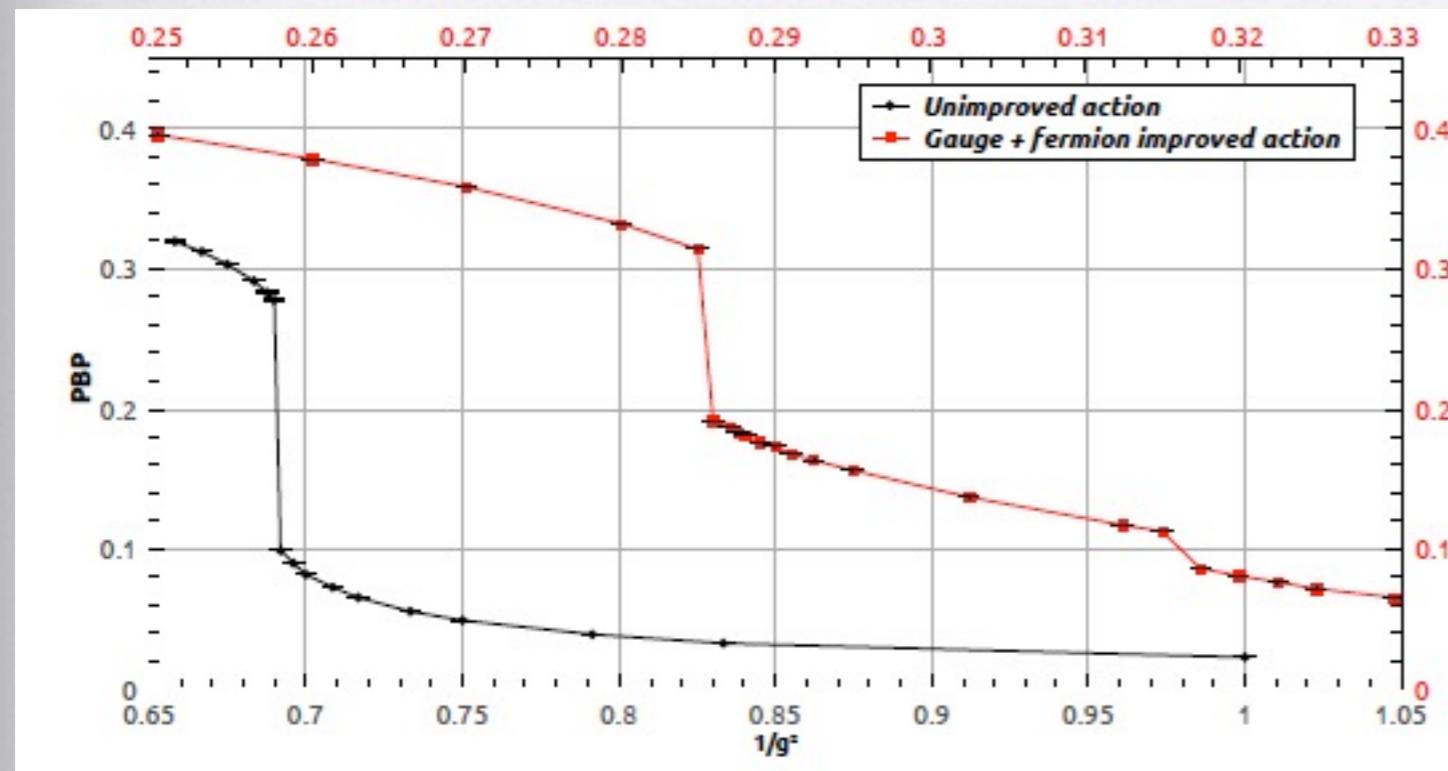
# Weakly chiral symmetry broken phase



Chang, Hasenfratz and Schaich:  
Phys.Rev.D85 (2012) 094509

HYP smearing

two jumps of chiral condensate and  
in the intermediate region shift  
symmetry is broken.



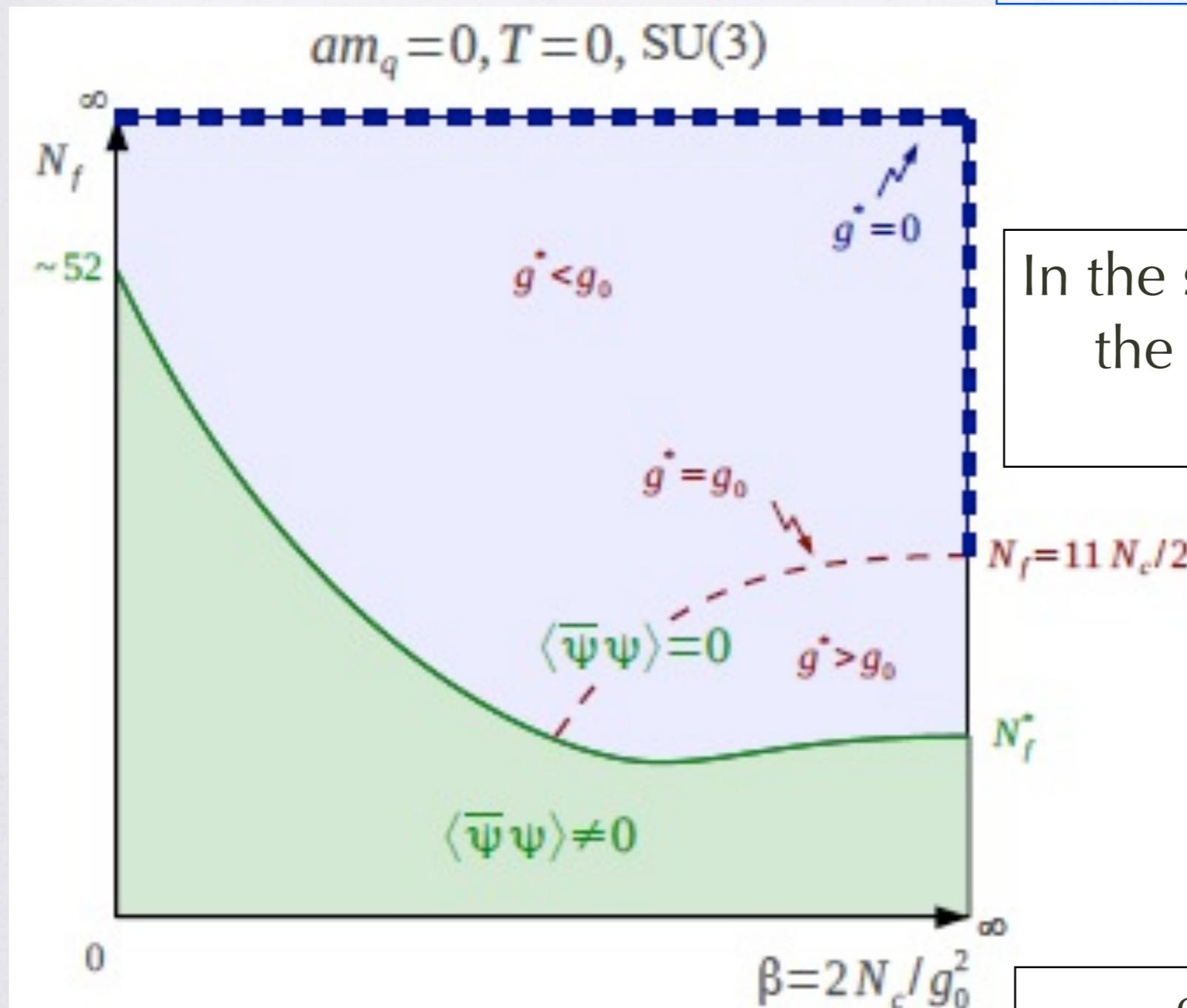
Deuzeman, Lombardo, da Silva and  
Pallante:  
arXiv: 1209.5720

Naik improvement

next-to-nearest neighbor terms  
are no longer irrelevant  
and indeed modify the pattern  
observed without improvement.

# conjectured phase diagram for many flavor SU(3) gauge theory

de Forcrand, Kim and Unger:  
arXiv:1208.2148



In the strong coupling limit,  
the chiral symmetry is  
broken  $N_f < 52$ .

cf. for  $\text{SU}(N_c)$ ,  
Tomboulis arXiv:1211.4842

# Our result

arXiv:1212.1353 [hep-lat]

# Simulation detail

Hybrid Monte Carlo algorithm

Wilson gauge action+ naive staggered fermion

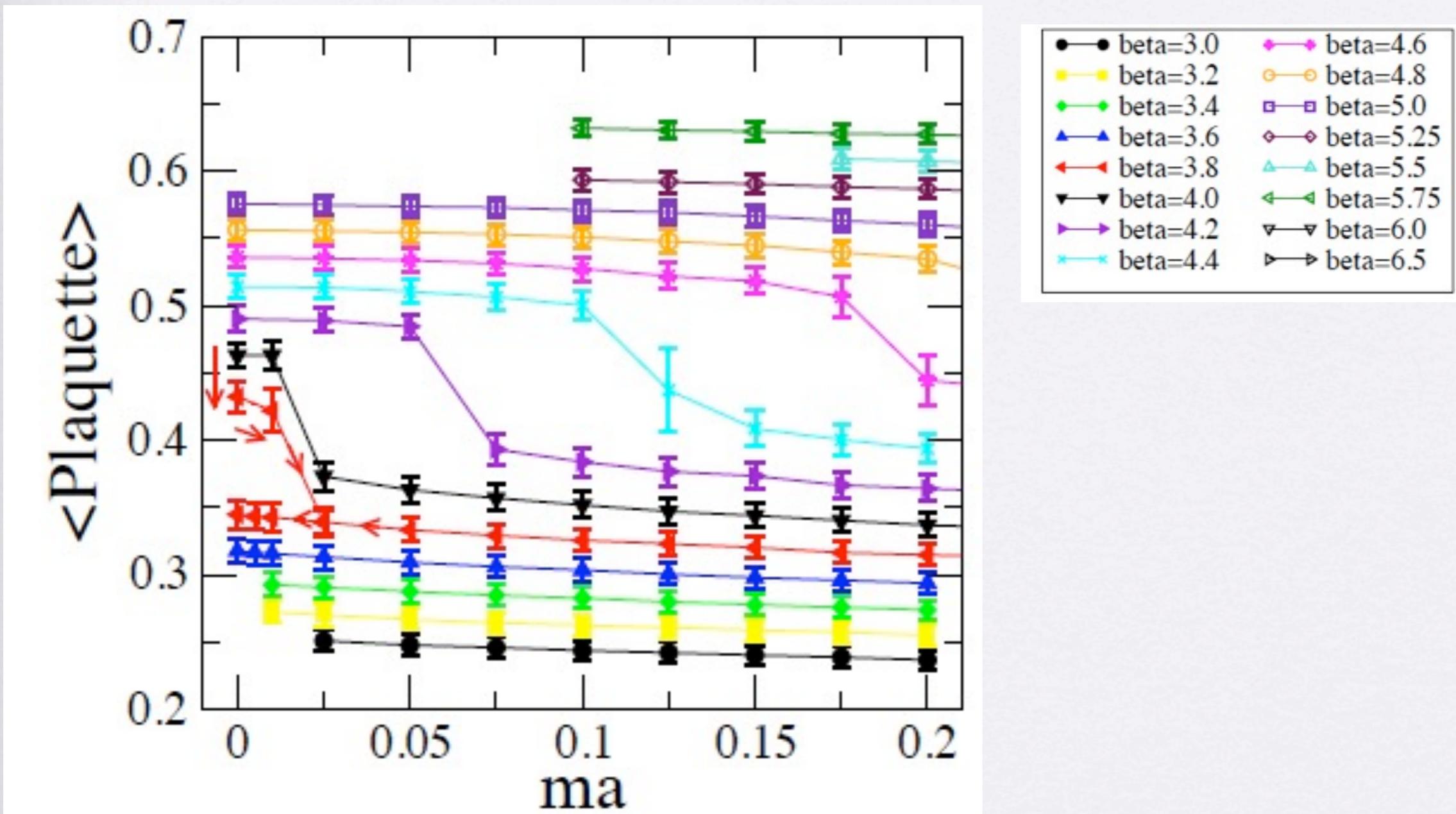
Twisted boundary condition for x,y directions

$\beta=4.0\text{--}100$  on  $(L/a)^4$  lattice

$L/a=6,8,10,12,16,20$

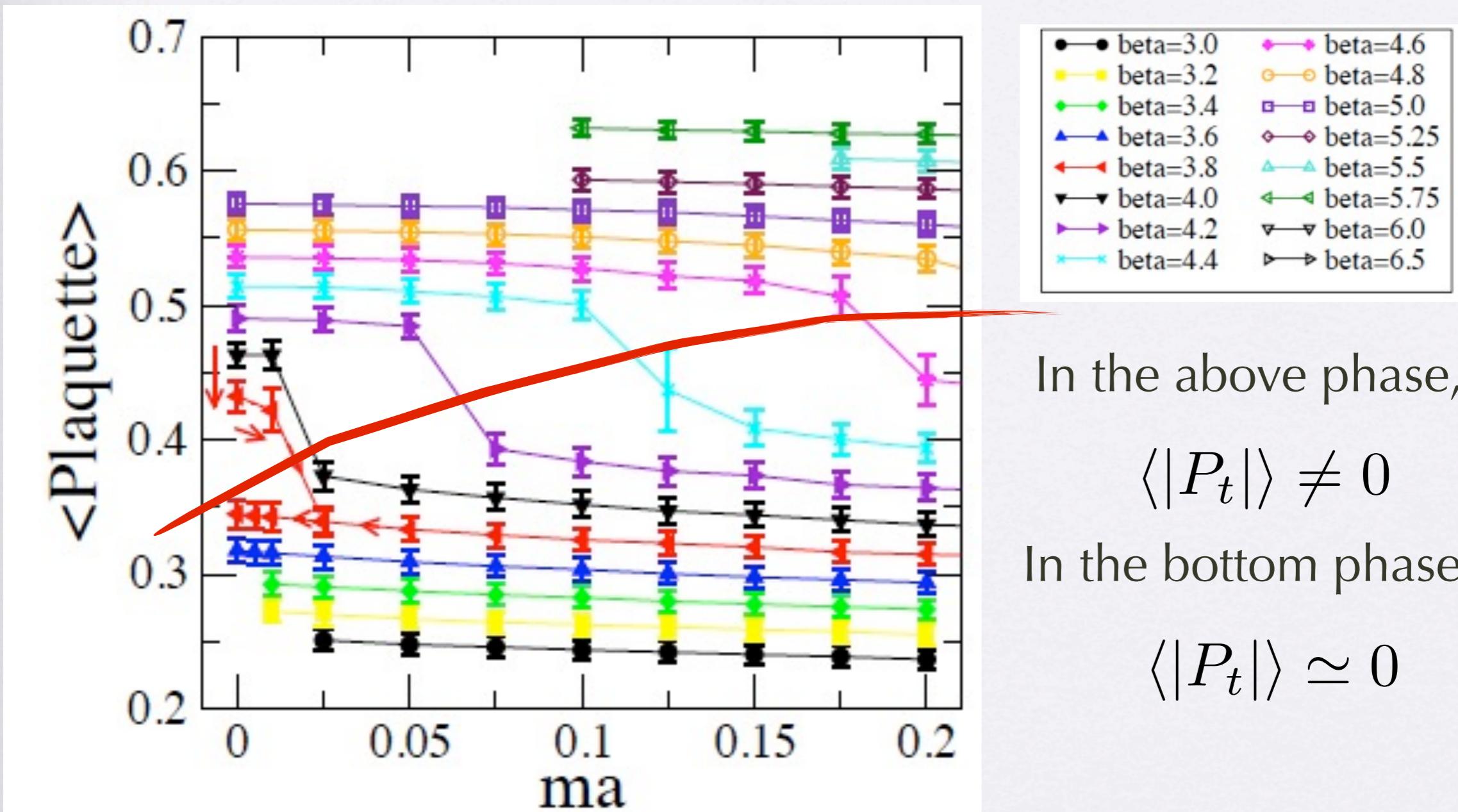
# Phase diagram in the lattice setup

In our simulation set up,  
there is a bulk phase transition in small mass region.



$$(L/a)^4 = 4^4$$

In our simulation set up,  
there is a bulk phase transition in small mass region.



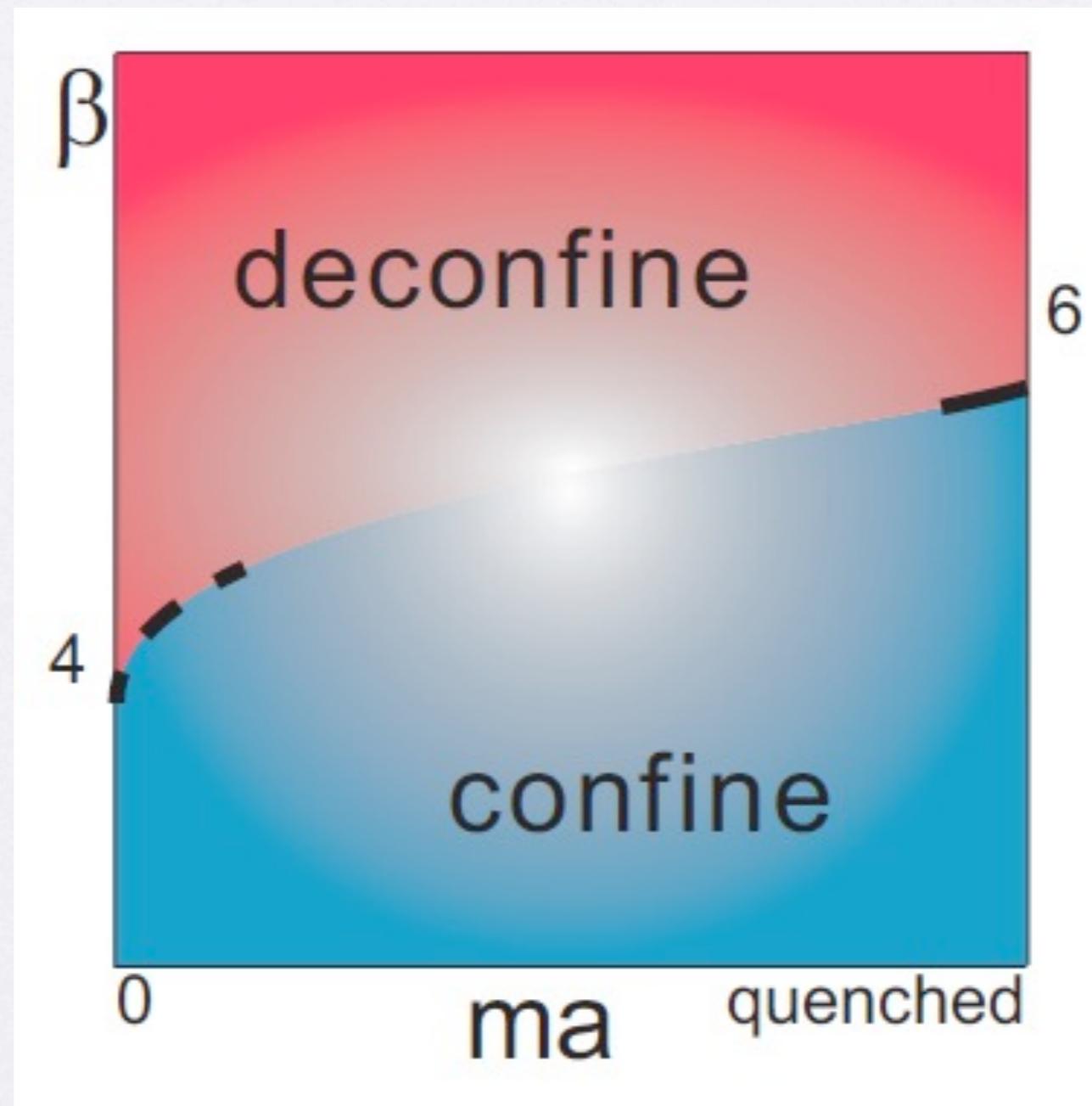
In the above phase,

$$\langle |P_t| \rangle \neq 0$$

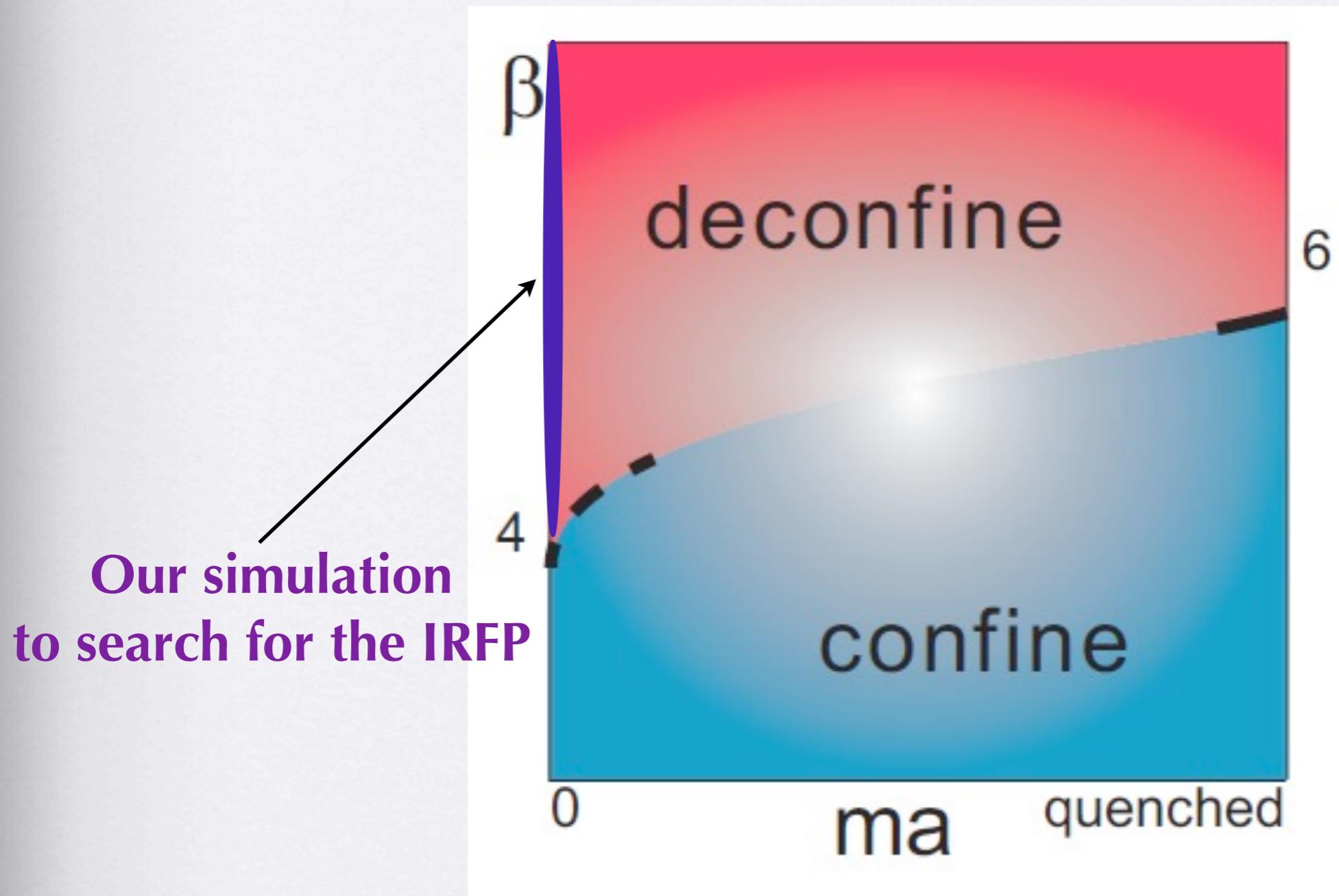
In the bottom phase,

$$\langle |P_t| \rangle \simeq 0$$

Phase diagram for SU(3) Nf=12 naive staggered fermion with the twisted boundary condition.



# Phase diagram for SU(3) Nf=12 naive staggered fermion with the twisted boundary condition.



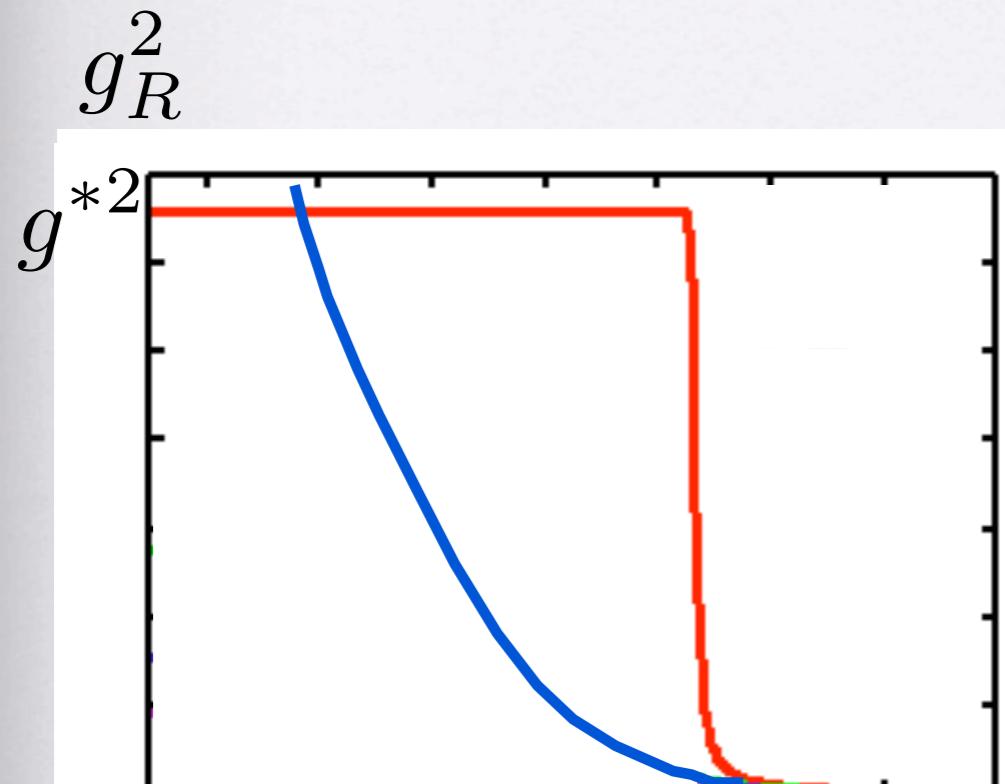
We also see that the chiral symmetry is preserved in this region.

# Running coupling

# Measure the growth ratio

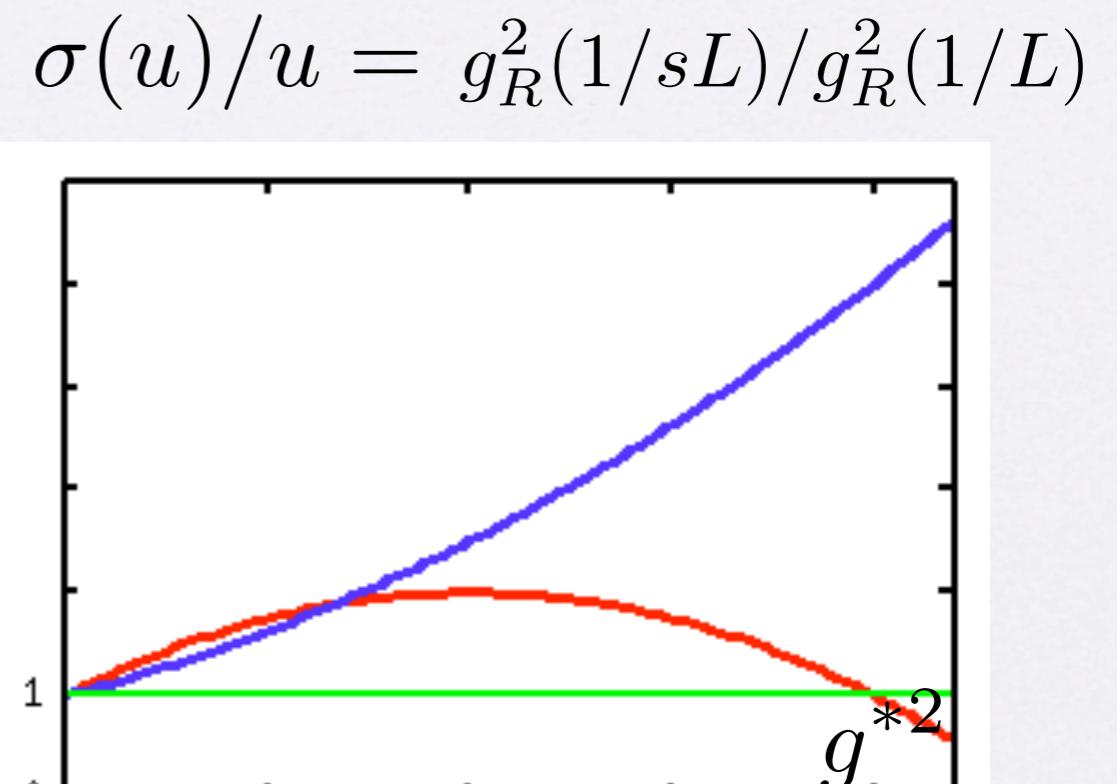
Observe the growth ratio of renormalized coupling constant  
to see the precise running behavior.

running coupling constant



$$\ln(L_0/L)$$

growth ratio



$$g_R^2(\mu = 1/L)$$

systematic error is accumulated

systematic error is not accumulated

# No O(a) renormalization scheme

A definition of nonperturbative renormalized coupling

$$\begin{aligned}\langle O \rangle_{NP} &\equiv Z_O \langle O \rangle_{tree} \\ &= Z_O g_0^2 \cdot k \\ &\equiv g_R^2 k\end{aligned}$$

Lattice simulation can calculate the vev of  $\langle O \rangle_{NP}$

$$g_R^2 \equiv \langle O \rangle_{NP} / k$$

Examples of renormalization scheme

Schroedinger functional scheme

Wilson loop scheme [Phys.Rev.D80:034507\(2009\)](#)

Twisted Polyakov Loop scheme

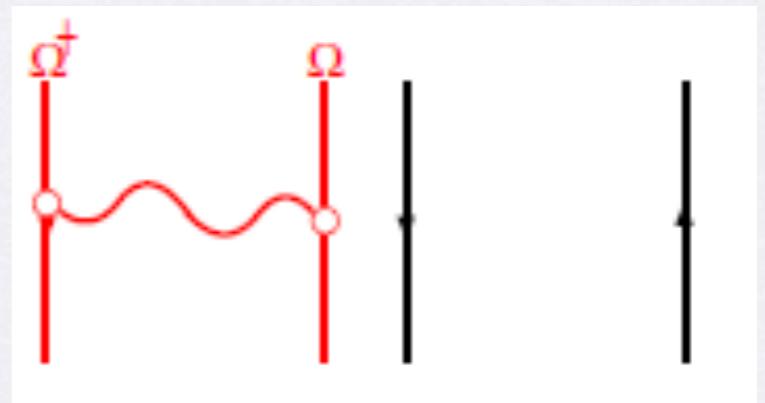
Wilson flow scheme....

} **no  $O(a/L)$  error scheme**

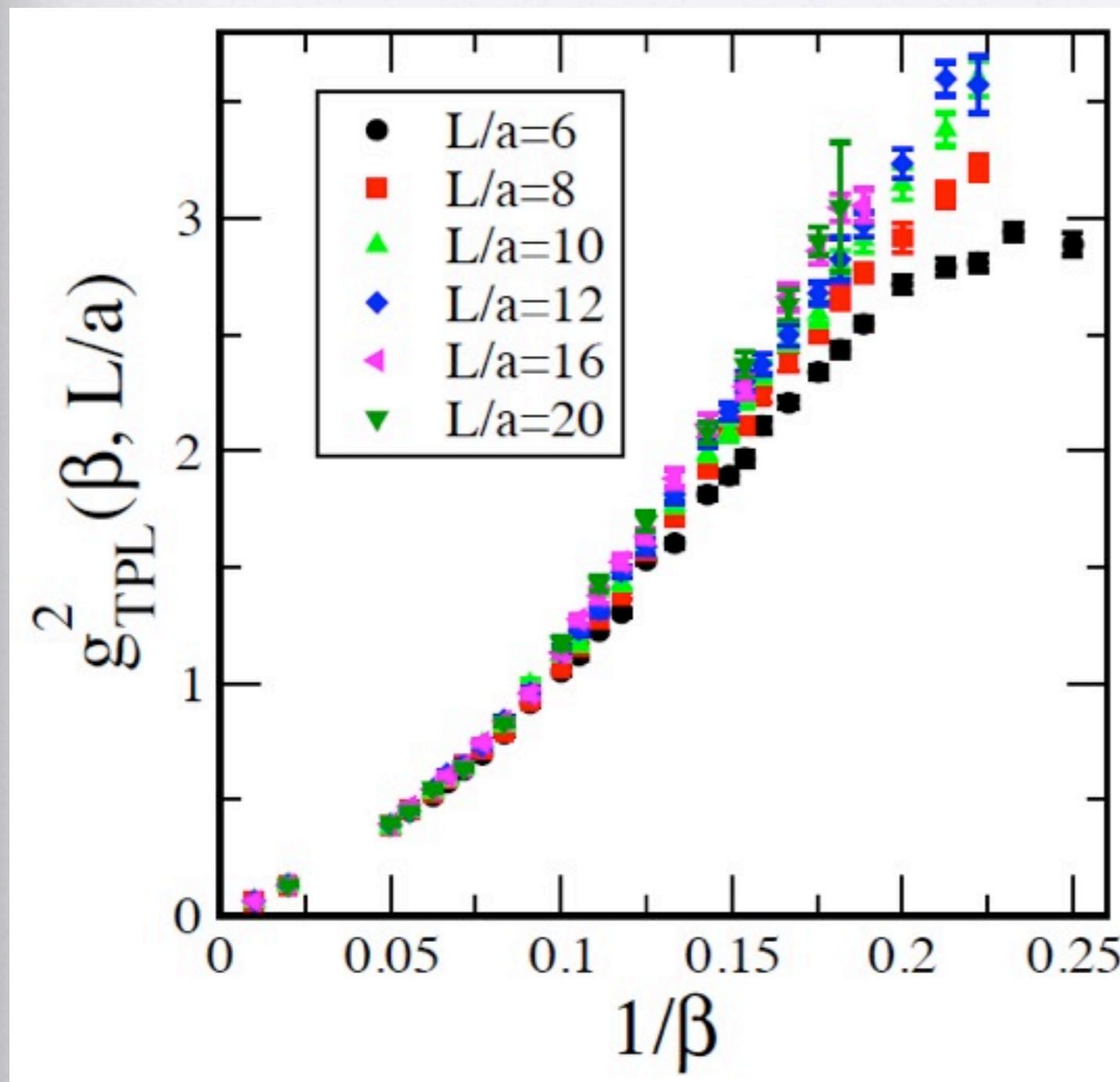
Nonperturbative definition of renormalized coupling in  
**Twisted Polyakov loop (TPL) scheme**

de Divitiis, Frezotti, Gaugnelli and Petronzio,  
NPB422(1994)382

$$g_{TP}^2 = \frac{1}{k} \frac{\langle \sum_{y,z} P_1(y,z,L/2a) P_1(0,0,0)^* \rangle}{\langle \sum_{x,y} P_3(x,y,L/2a) P_3(0,0,0)^* \rangle}$$



# Raw data in TPL scheme



**2-3 % statistical error.**

# of Trj is 64,400- 1,892,800.

**Fitting fn. for beta interpolation**

$$g_{TPL}^2(\beta, L/a) = \frac{6}{\beta} + \sum_{j=1}^N \frac{C_j(L/a)}{\beta^{j+1}}$$

**s=1.5 step scaling**

$L/a=6 \rightarrow L/a=9$

$L/a=8 \rightarrow L/a=12$

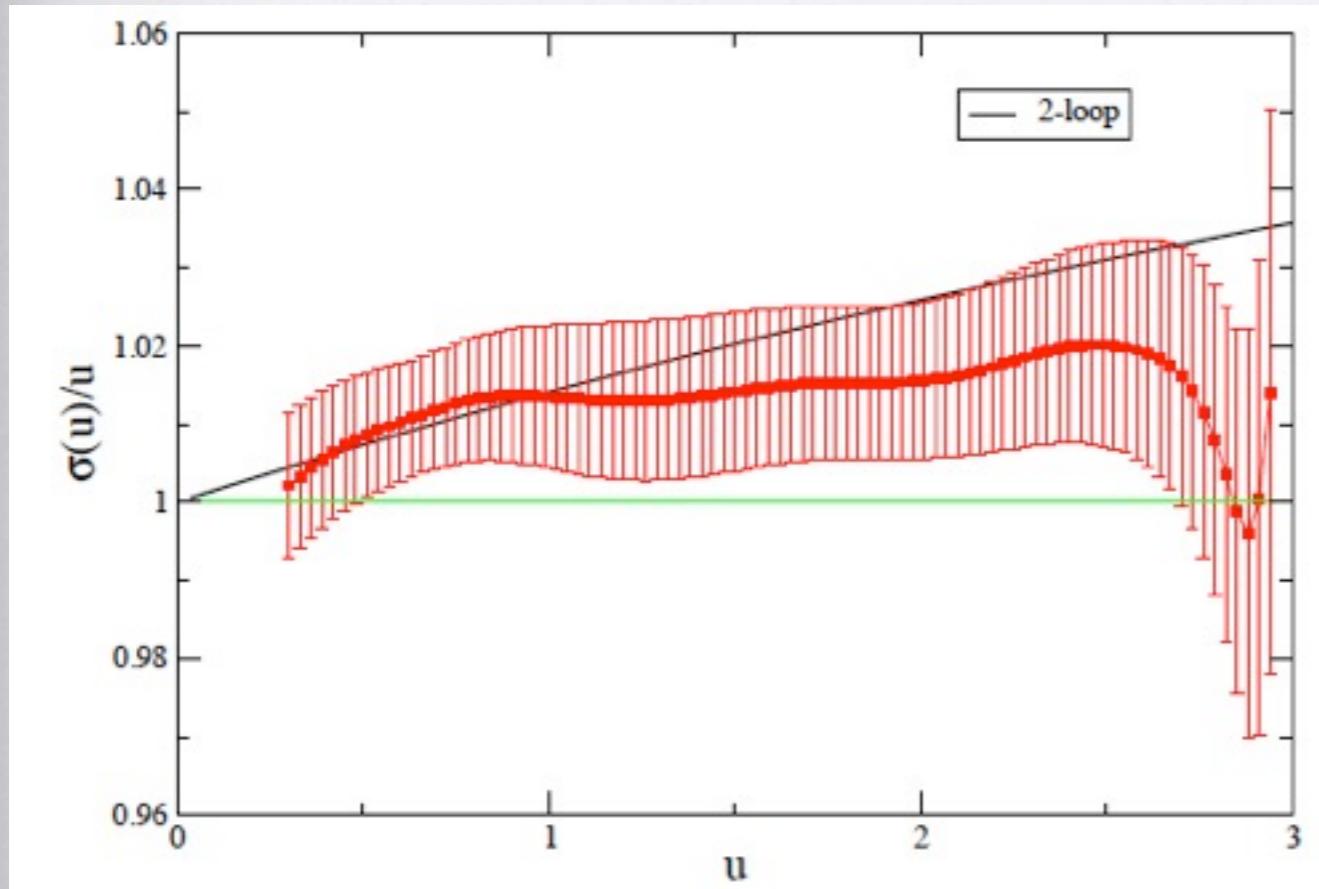
$L/a=10 \rightarrow L/a=15$

$L/a=12 \rightarrow L/a=18$

For  $L/a = 9, 15$  and  $18$ ,  
we estimate values of  $g_2$  for a given  $\beta$   
by the linear interpolation in  $(a/L)^2$ .

$$1/\beta (= g_0^2/6)$$

# Growth ratio of TPL coupling (global fit analysis)



TPL coupling shows the fixed point around

$$g_{\text{TPL}}^{*2} \sim 2.7$$

This is the first zero point of the beta function from the asymptotically free region, it must be IR fixed point.

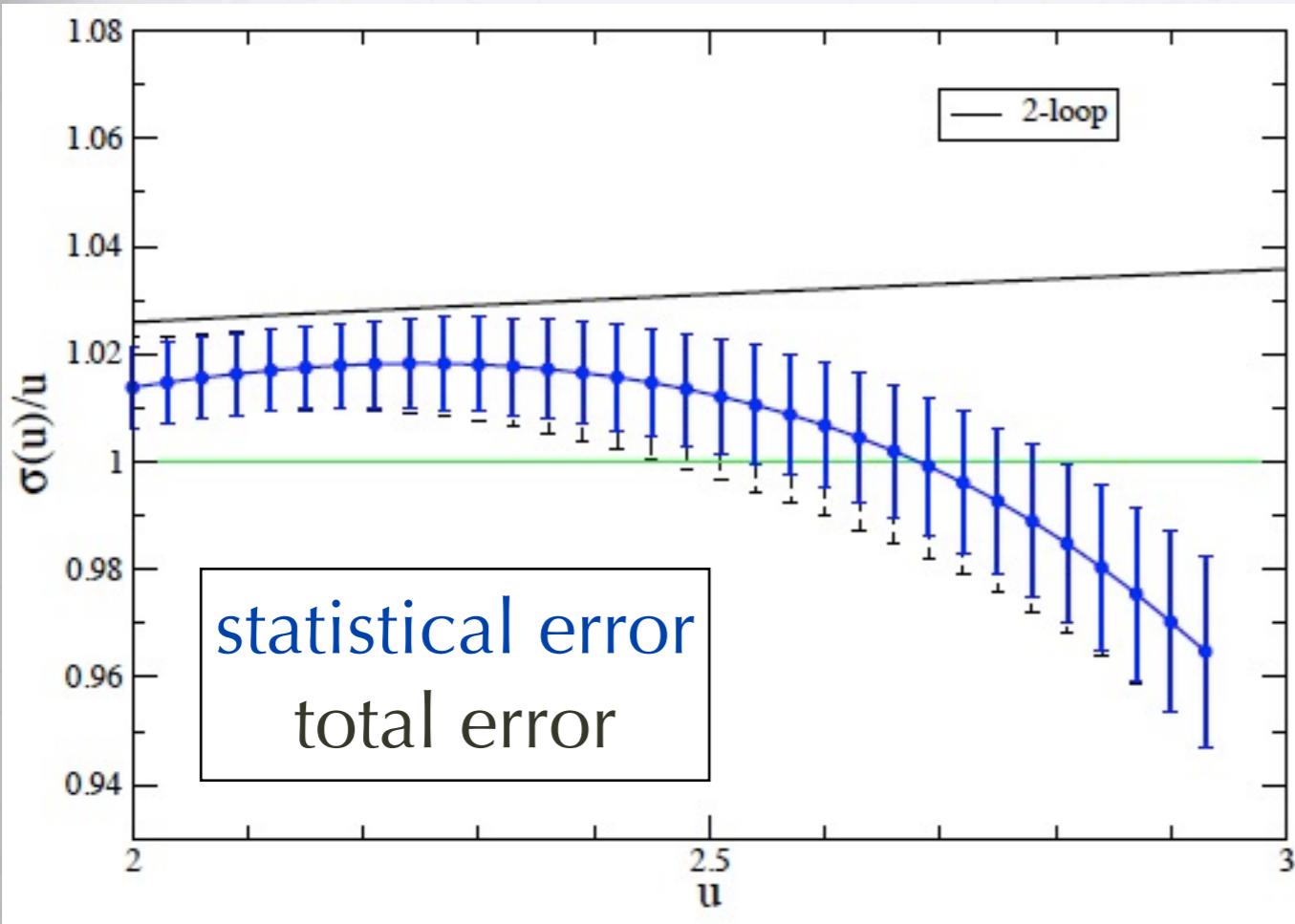
Unfortunately, the growth ratio with errorbar does not cross over the unity line.

## Local fit analysis

Focus on the low beta region ( $\mu > 2.0$ )

Add the data (more than 30 data points)

# Growth ratio of TPL coupling (local fit analysis)



$$g_{\text{TPL}}^{*2} = 2.69 \pm 0.14(\text{stat.})^0_{-0.16}(\text{syst.})$$

Around the fixed point, the beta fn. can be approximated by the linear fn.

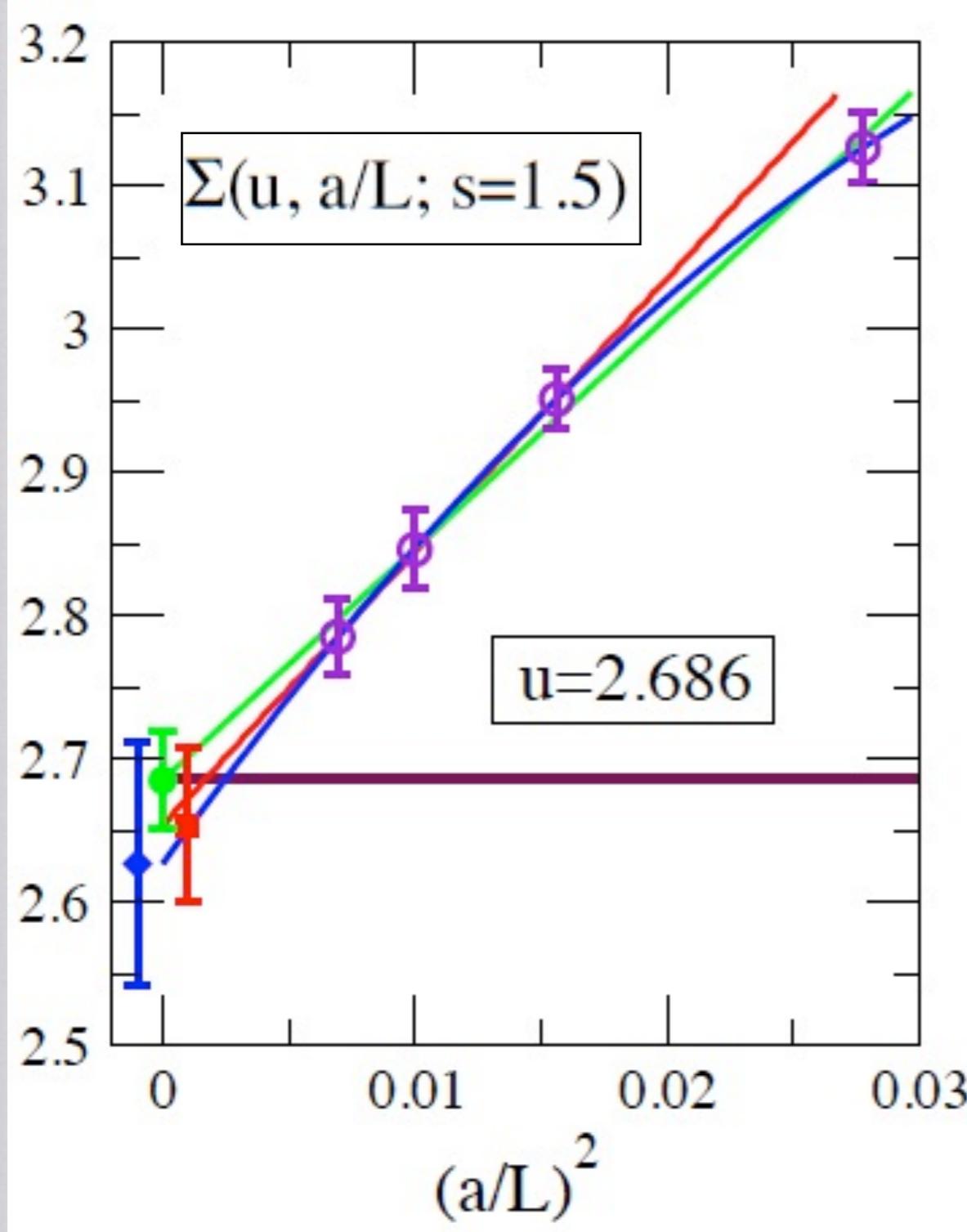
$$\beta(g^2) \sim \gamma_g^*(g^{2*} - g^2)$$

Our result

$$\gamma_g^* = 0.57^{+0.35}_{-0.31}(\text{stat.})^0_{-0.16}(\text{syst.})$$

SF scheme	2 loop at $g^{2*} = 9.4$	4 loop (MS bar)
$\gamma_g^* = 0.13 \pm 0.03$	$\gamma_g^* = 0.36$	$\gamma_g^* = 0.28$

# Continuum extrapolation



**s=1.5 step scaling**

$L/a=6 \rightarrow L/a=9$

$L/a=8 \rightarrow L/a=12$

$L/a=10 \rightarrow L/a=15$

$L/a=12 \rightarrow L/a=18$

2 loop prediction  
in this region is  
 $\sigma(u = 2.69) \sim 2.78$

The systematic error is small in the strong coupling region.

# Is there an IR fixed point in SU(3) Nf=12 theory?

Iwasaki et al, '04 '13 (phase structure)

Appelquist, Fleming, Neil '07, '09, '11 (running coupling, mass spectrum)

Deuzeman, Lombardo, Pallante, Miura '09, '11(finite temperature)

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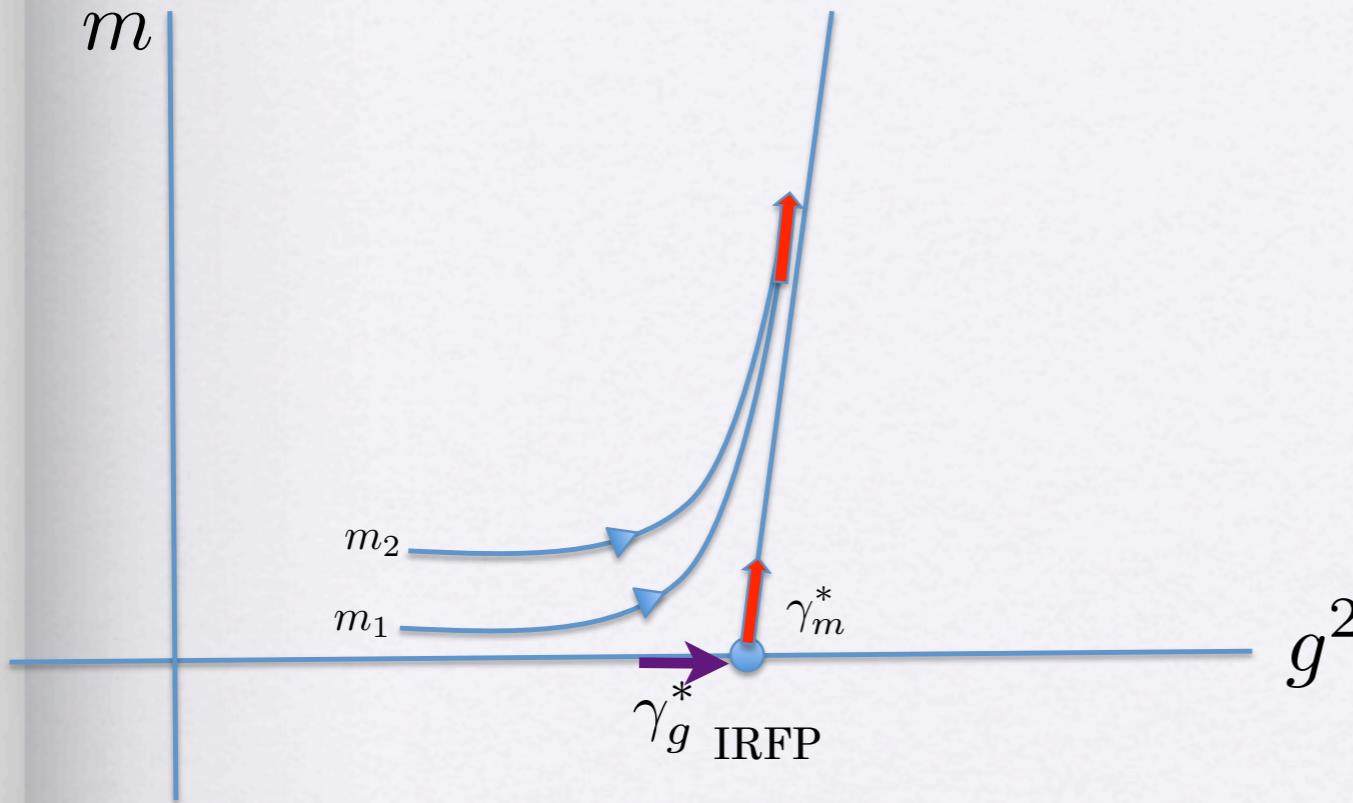
Jin and Mawhinney '09 (phase structure)

YES

# Anomalous dimension

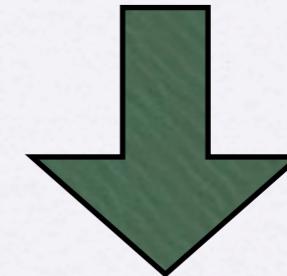
## —Preliminary result—

# The other important critical exponent around the IRFP



2-quark and 2-techni-fermion

$$\frac{c_2}{\Lambda_{ETC}^2} \langle \bar{\Psi} \Psi \rangle_{ETC} (\bar{\psi} \psi)$$



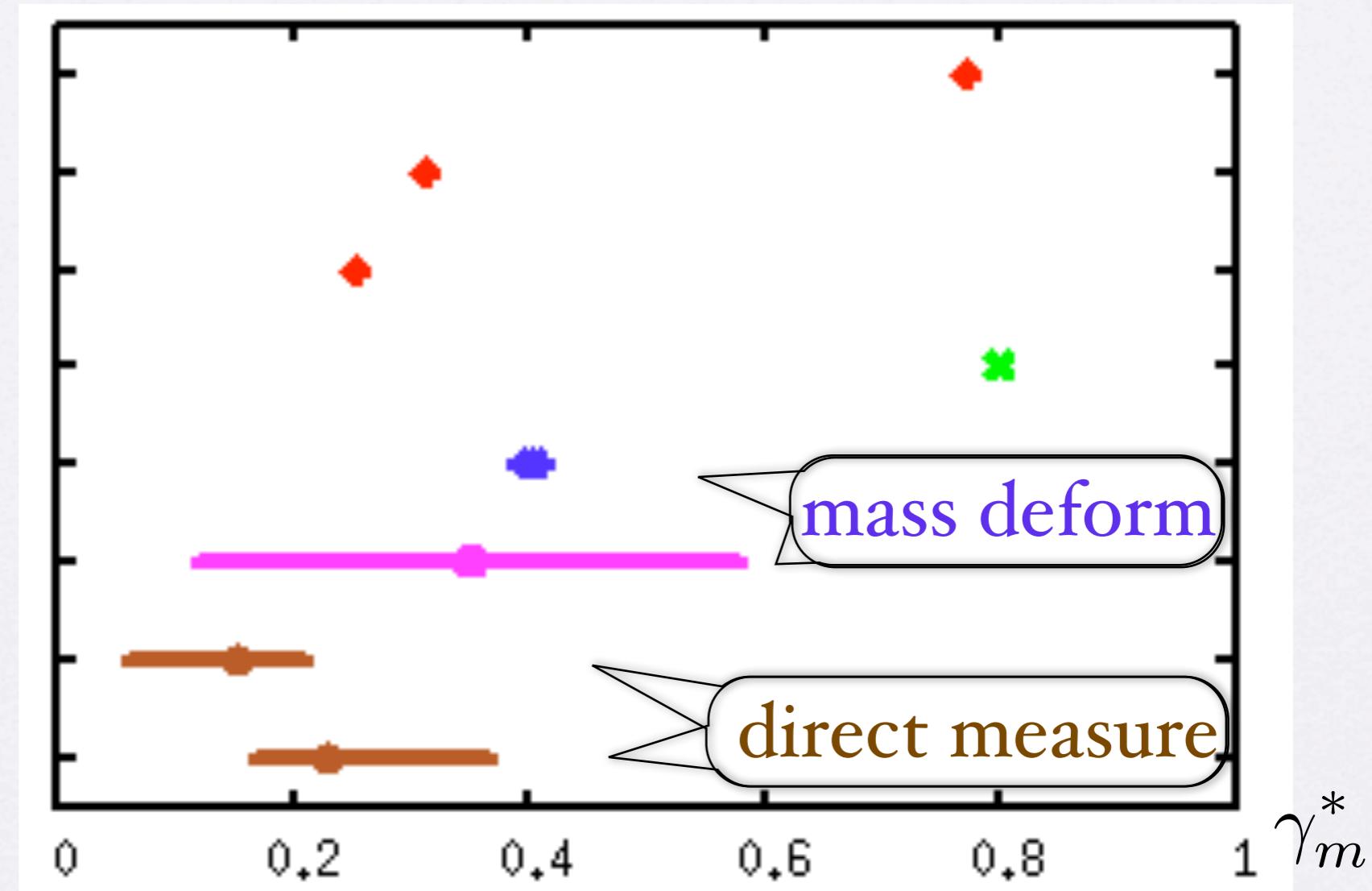
$$M_q \sim \frac{1}{\Lambda_{ETC}^2} \left( \frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma^*} \langle \bar{\Psi} \Psi \rangle_{ETC}$$

2loop prediction:

$$\gamma_g^* = 0.36 \quad \gamma_m^* = 0.77$$

# Mass anomalous dim from the several studies

2 loop  
3 loop (MS bar)  
4 loop (MS bar)  
Schwinger-Dyson  
(ladder)  
Appelquist et.al  
DeGrand  
Our result ( $r=1/3$ )  
( $r=1/4$  w. stat. err.)  
(preliminary)



# correlation fn. of nearly conformal theory

Ishikawa, Iwasaki, Nakayama and Yoshie: arXiv:1301.4785

two point fn. of a meson state

$$G_H(t) = \sum_x \langle \bar{\psi} \gamma_H \psi(x, t) \bar{\psi} \gamma_H \psi(0) \rangle$$

conformal theory (massless, continuum)  $G_H(t) = \tilde{c} \frac{1}{t^{\alpha_H}}$   $\alpha_H = 3 - 2\gamma^*$

confined theory

$$G_H(t) = c_H \exp(-m_H t)$$

(nearly) conformal theory

$$G_H(t) = \tilde{c}_H \frac{\exp(-\tilde{m}_H t)}{t^{\alpha_H}}$$

To extract the mass spectrum, we have to use the Yukawa-type fit fn.

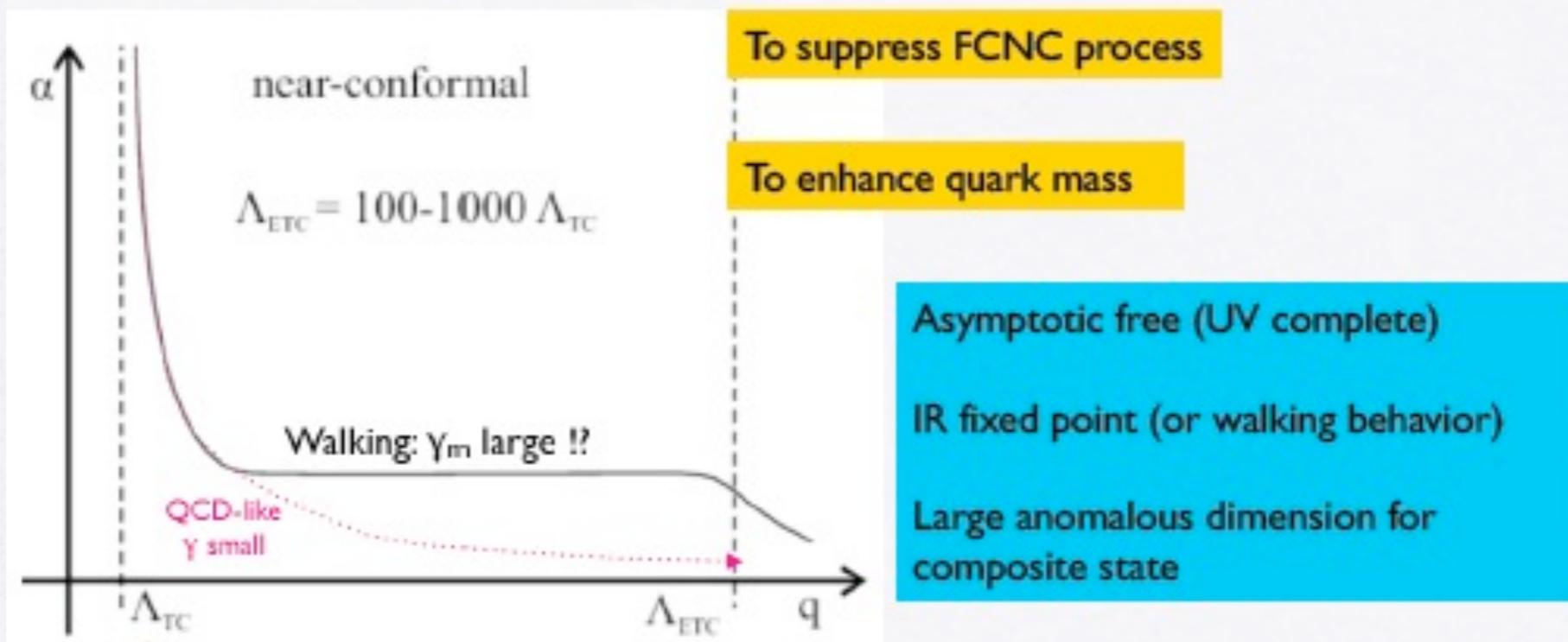
# Conclusion and Discussion

There is the IRFP in SU(3) Nf=12 massless theory.  
Continuum extrapolation and parameter search are important.

- (1) The phenomenological model construction by using the mass anomalous dimension from the lattice simulation.
- (2) Derive the universal quantities (anomalous dimension) precisely
- (3) Spectrum for several modes, who is the lightest state?  
pseudoscalar? dilaton?
- (4) We have to understand the property of conformal field theory on the finite lattice volume.

Lattice precise data can give phenomenological and theoretical information around nontrivial fixed point.

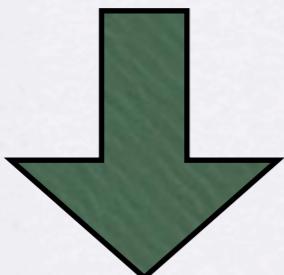
# phenomenological prediction?!



Assume that  $\Lambda_{TC} \sim 1\text{TeV}$ ,  $\Lambda_{ETC} \sim 1000\text{TeV}$

2-quark and 2-techni-fermion

$$\frac{c_2}{\Lambda_{ETC}^2} \langle \bar{\Psi} \Psi \rangle_{ETC} (\bar{\psi} \psi)$$



$$M_q \sim \frac{1}{\Lambda_{ETC}^2} \left( \frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma_m^*} \langle \bar{\Psi} \Psi \rangle_{ETC}$$

$\gamma_m^* = 1$	$M_q \sim 1\text{GeV}$
$\gamma_m^* = 0.5$	$M_q \sim 30\text{MeV}$
$\gamma_m^* = 0.25$	$M_q \sim 5\text{MeV}$
$\gamma_m^* = 0.15$	$M_q \sim 2\text{MeV}$